2 special limits "squeeze theorem"

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x\to 0} \frac{\cos x - 1}{x} = 0}$$

Q: Derivatives of trig functions?

① $$\frac{d}{dx}[\sin x] = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{(\sin(x)\cos(h) + \cos(x)\sin(h)) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{(\sin(x)\cos(h) - \sin(x)) + \cos(x)\sin(h)}{h}$$

$$= \lim_{h\to 0} \left(\sin(x)\left(\frac{\cos(h)-1}{h}\right) + \cos(x)\frac{\sin(h)}{h}\right)$$

$$= \sin(x)\lim_{h\to 0}\frac{\cos(h)-1}{h} + \cos(x)\lim_{h\to 0}\frac{\sin(h)}{h}$$

$$= \sin(x)\cdot 0 + \cos(x)\cdot 1$$

$$= \cos x$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\boxed{\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \end{aligned}}$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

Need quotient rule!

__Quotient Rule__: If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Ex: ① $$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

$$= \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

② $$\frac{d}{dx}\left[\frac{e^x + 3\sin x}{\sqrt{x} - 4\cos x}\right]$$

$$= \frac{(e^x + 3\cos x)(\sqrt{x} - 4\cos x) - (e^x + 3\sin x)(\frac{1}{2}x^{-1/2} + 4\sin x)}{(\sqrt{x} - 4\cos x)^2}$$

③ $$\frac{d}{dx}[\cot x] = -\csc^2 x$$

④ $$\frac{d}{dx}[\sec x] = \sec x \tan x$$

⑤ $\frac{d}{dx}\left[\frac{x^2+x-2}{x^3+6}\right] = \frac{(2x+1)(x^3+6)-(x^2+x-2)\cdot 3x^2}{(x^3+6)^2}$

⑥ $\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{0\cdot\sin x - 1\cdot\cos x}{\sin^2 x}$

$= \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}$

$= -\cot x \csc x$

$\frac{d}{dx}[x^n] = nx^{n-1}$, $\frac{d}{dx}[e^x] = e^x$

$\frac{d}{dx}[\sin x] = \cos x$, $\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\tan x] = \sec^2 x$, $\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$, $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

$\frac{d}{dx}[cf(x)] = cf'(x)$

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$